



Research and Development Report

TELEVISION COLORIMETRY: A tutorial for system designers

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Summary

This Report was written primarily for use as a colorimetry tutorial for any MPEG users who may feel confused by the plethora of descriptive data contained in the MPEG data-stream. It is not intended to be an exhaustive treatise on basic colorimetry or even on television colorimetry; that would require a very much larger document. There are many books on the subject, but two are of particular relevance, those by Sproson,¹ and Widdel and Post.²

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1. INTRODUCTION

Little has changed in television colorimetry since the first colour broadcasts began, until recent times. Early colour coding systems were always designed such that decoding could be done for lowest cost, and this was the most important feature of the system, provided that performance was adequate. With the advent of digital coding and transmission systems this situation is changing rapidly. It is, therefore, now appropriate to re-examine the principles of colour picture coding to see, not only how improvements might be made, but whether the best performance is being achieved within the constraints of the existing systems.

The proposals for ISO/IEC MPEG II digital video coding form a good example of the ways in which picture coding is changing, since it is planned to include, in the data stream, codes which identify many of the parameters used in the original picture encoding. Whilst laudable in itself, this proposal places a burden of understanding on the end user of the transmitted information who must properly interpret the meaning and significance of each data word, if full and accurate use of the information is to be achieved with the best resultant picture quality. In this context, the old adage *“a little learning is a dangerous thing”* could easily be extended into *“a little learning may be dangerous but an excess of information can be bewildering”*. The problem exists primarily because all the source parameters can be specified independently; so it is easy to use wrong combinations without realising what has happened. For example, the proposal contains separate codes to specify the ‘camera primaries’, the opto-electronic transfer function, and the signal coding equations. In doing so, the proposals cover all the commonly used coding systems, and allow a large capacity for expansion. Thus, there is plenty of opportunity for confusion, but provided that the user clearly understands how all this information can be used, there should be no problem.

2. HOW VIDEO SYSTEMS AND TELEVISION WORK

This section briefly describes how the colorimetry of conventional picture origination and display actually works. It provides the reasoning behind the use of the parameters carried in the MPEG coding data. MPEG users wishing to be certain that their use of the MPEG

data is colorimetrically correct should read this section.

There are three sets of parameters which define (in colorimetric terms) the coded signal:

- The ‘Camera Primaries’
- Opto-electronic transfer characteristic (gamma curve)
- Coding equations

2.1 ‘Camera primaries’

Strictly, cameras do not have primaries; they have ‘taking characteristics’ which are the colour-matching functions* of the primaries of the display upon which the camera pictures are intended to be displayed. In practical terms, this means the primaries of the monitor in the studio control room; it is there that the engineers adjust the camera controls to produce an acceptable picture. For example, in Europe, the display has the EBU primaries,³ with Red specified** as $x = 0.64$, $y = 0.33$, Green as $x = 0.29$, $y = 0.6$, and Blue as $x = 0.15$, $y = 0.06$; while in ITU-R Rec. BT. 709 they are the same Red and Blue, but Green is $x = 0.3$, $y = 0.6$. The colour of the white-balance point is also important (the colour for which the three drive signals are equal to each other), and is usually taken to be D_{65} , an average colour of daylight.

The ideal camera taking-characteristics are the colour-matching functions of these primaries. Wavelength-by-wavelength through the spectrum, the camera should produce the *RGB* voltages (linear) which would generate light of that wavelength on the prescribed monitor. Unfortunately, this always requires the camera to produce negative *RGB* values (and that implies the display ‘absorbing light’ which is clearly impossible), but the principle is sound and forms part of the current colorimetric proposal for digital HDTV in North America. The ideal characteristics for an ITU-R Rec. BT. 709 compliant camera are shown in

* Colour-matching functions are spectral responsivity curves rather than emissivity (or reflectivity). They are related to the colour vision of the average, colour-normal, human observer. See Appendices I and II for details.

** The values of x and y are chromaticity coordinates, which describe the colour but ignore the luminance. See Appendix I for details.

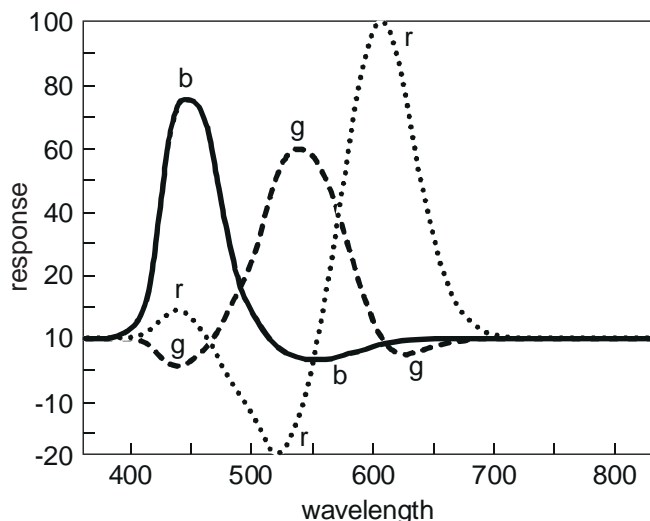


Fig. 1 - Ideal ITU-R Rec. BT. 709 camera characteristics.

Fig. 1. The taking characteristics of practical cameras are deliberately distorted from this ideal in the interests of sensitivity, ease of manufacture, and the long held desire for non-linear colorimetry in the TV system.

For each set of ‘camera primaries’ (i.e. each set of studio monitor primaries) there is an ideal set of equations* which relate the scene colorimetry in CIE XYZ** tristimuli to the camera *RGB* tristimuli. The linear matrices which define this relationship are **unique** and can be used in colour-matching identities:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

and its inverse

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} R_X & R_Y & R_Z \\ G_X & G_Y & G_Z \\ B_X & B_Y & B_Z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

The matrix coefficients are the tristimuli of the right-hand-side primaries in terms of the left-hand-side primaries. The balance condition of $R = G = B = 1$ happens at the white point (e.g. D_{65}), which is also specified, and this affects the matrix values. At the white point:

* These equations are definitive for each set of primaries. See Appendix II for details.

** The CIE 1931 XYZ system is used universally for colour specification and measurement. The Y primary in this system is defined as luminance. See Appendix I for details.

$$\begin{bmatrix} X_{D_{65}} \\ Y_{D_{65}} \\ Z_{D_{65}} \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X_R + X_G + X_B \\ Y_R + Y_G + Y_B \\ Z_R + Z_G + Z_B \end{bmatrix}$$

and the solution to this equation provides the values for the matrices by straightforward arithmetic. Note that the centre line of the first matrix **defines** the **only** correct luminance equation for these primaries, and that this luminance value is the true ‘grey-scale’ brightness of a colour thus specified.

Note also that these are **ideal** equations which assume that the camera has ideal taking characteristics. In practice, the curves may be distorted from the ideal for many reasons, and the matrix equations are therefore only approximate statements of relationships in real cameras. However, the equations are truly representative of the performance of picture monitors, provided that non-linearities are taken into account as shown in the following sub-section.

2.2 Transfer characteristics

The camera primary signals are pre-distorted for three reasons:

- Historically, it has always been believed that the overall system should have a transfer characteristic (light-in to light-out) which obeys a power law of rather more than unity. Values of about 1.2 to 1.3 are typical. This makes the displayed scene seem rather larger than life, and to some extent compensates for viewing with a darkened surround. In practice, this ‘power law’ is only an approximation since the camera does not have a true power law (see below).
- The main real-time display for moving pictures has always been the cathode ray tube, with a power law of between 2 and 3. Recent measurements using improved techniques have shown that colour CRTs have transfer characteristics with a power law of between 2.3 and 2.4, and that this law is maintained all the way from peak brightness to black, there is no significant deviation from the power law.⁴ To keep life simple, the camera pre-distorts the *RGB* signals in an attempt to match the display. This results in signals which can be satisfactorily monitored in the studio using a very simple monitor with direct *RGB* drives.

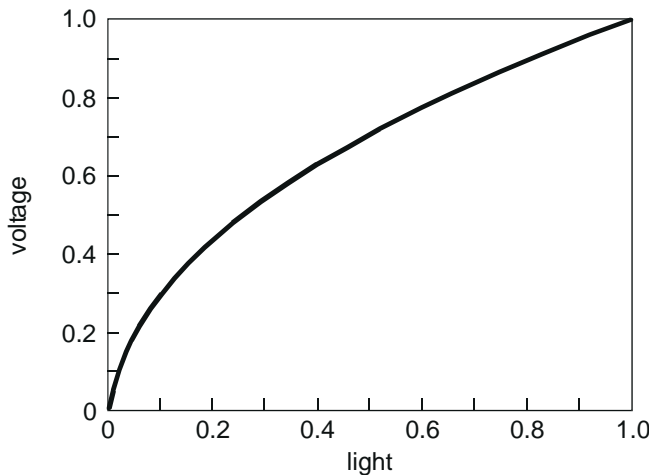


Fig. 2 - ITU-R Rec. BT. 709 camera transfer characteristic.

- Transmission through a noisy (analogue or highly compressed digital) environment adds noise to the signals (quantisation and compression artefacts can be regarded as noise for this purpose). A power law distortion helps to equalise the visibility of this noise over the luminance scale. The eye responds to light approximately logarithmically* and so an approximately logarithmic pre-distortion of the luma** signal helps to minimise the visibility of noise.

Two types of correction equation, relating camera output voltage V to *Light*, are widely in use:

$$V = (1 + a) \text{Light}^\gamma - a \quad \text{and} \quad V = \left(\frac{\text{Light} - b}{1 - b} \right)^\gamma$$

where a and b are constants which move and scale the power law along the *Light* or V axis respectively, away from zero. Both types of equation produce curves which allow for a straight line portion at low levels, such that the gain at black does not have to be infinite (as is required in a true power law equation). Careful choice of offsets and breakpoint (from the exponential into the linear portion) ensures that the transition from power law to linear is at best tangential, or at least

* There is an approved formula for the response of the eye, specified by the CIE. In television terms it can be defined as $V = 1.16L^{1/3} - 0.16$. At a very low brightness the relationship is linear, with a slope of about 9. It approximates to a conventional television equation with gamma about 0.42.

** The 'luma' signal carries luminance information and is conventionally termed the 'luminance' signal. The new term is introduced here to distinguish between what is seen (luminance) and what is transmitted (luma). Similarly, chroma is used to define the transmitted signals, but chrominance is used to define the perceived colour information (hue and saturation).

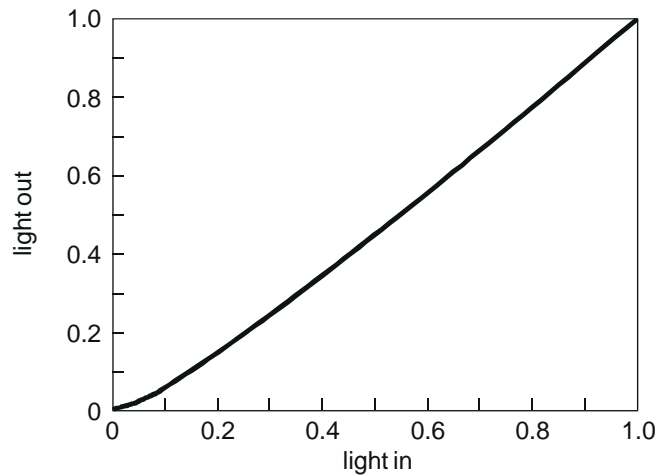


Fig. 3 - System curve, ITU-R Rec. BT. 709 camera plus CRT display.

smooth. Not all of the popular laws produce a tangential linear portion; those that do so result in more tractable colorimetry.

The commonly used equations are all compromises between:

- colorimetric accuracy (which demands a true power law)
- minimising the visibility of transmission noise (which also demands a true power law)
- minimising camera pickup device noise (which demands linearity to avoid magnifying noise at black).

The ITU-R Rec. BT. 709 curve is shown in Fig. 2, and the overall characteristic of a system using that camera curve and a cathode ray display with $\gamma = 2.35$ is shown in Fig. 3.

It is a common mistake to speak of the camera transfer law as a gamma curve. It is the display which has the power law (gamma), the camera has only an imperfect compensating law for the reasons listed above.

The effect of using an overall characteristic approximating to a power law of greater than unity is not just to give the picture more contrast; such systems produce 'poster-colour' pictures, where colours are generally pushed towards the display primaries (*RGB*) and secondaries (yellow, magenta, cyan). Quite small deviations from an overall linear response can result in quite large colour errors.

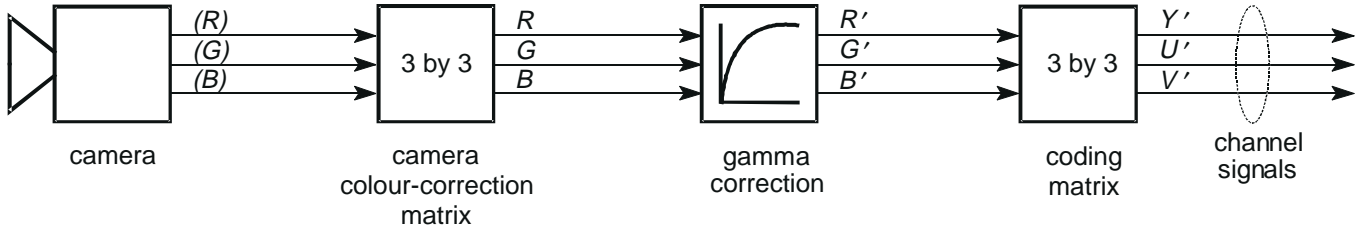


Fig. 4a - Conventional coder.

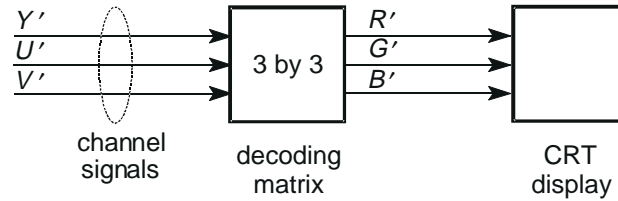


Fig. 4b - Conventional decoder.

2.3 Conventional coding

All practical systems generate a luma signal from non-linear (gamma-corrected) *RGB* signals, using proportions derived from a matrix equation as described earlier:

$$L' = Y_R R' + Y_G G' + Y_B B'$$

where the prime (') denotes gamma-correction. Two colour difference signals (chroma) are also generated:

$$U' = C_B = \alpha (B' - L') \quad \text{and} \quad V' = C_R = \beta (R' - L')$$

using factors α and β simply to scale them to fit the dynamic range of the transmission system.

Clearly, different luma and chroma equations should be used for every set of primaries specified. In practice (and for historical reasons), most systems generate the luma signal using the luminance proportions for the old NTSC primaries (which nobody now uses for real displays).

Fig. 4 shows a block diagram of a conventional coder and decoder. The precise nature of the transmission of these signals is not the concern of this document.

The luma signal is generally assumed to be orthogonal to planes of specified hue and chroma, but this is so only where it pierces them (at $R = G = B$, the achromatic or white point of the system). Since the luma signal is formed from a mixture of non-linear signals, it does not truly represent the *perceived* luminance which is represented by the centre line of the matrix equation using linear components:

$$Y = Y_R R + Y_G G + Y_B B$$

The difference between the gamma-corrected luminance value Y' and the transmitted luma value L' is carried in the chroma signal. Normally, the chroma signals are low-pass filtered or compressed and so the part of the luminance signal which does not travel via the luma channel is thus lost at high frequencies. This is called “failure to observe the constant-luminance principle”. When the chroma signals are bandwidth-limited, or compressed in any way, this use of conventional equations results in a luminance resolution loss which increases with saturation, and in some chroma channel noise being decoded and perceived as luminance noise. In fact, at any value of perceived luminance, the chroma lies not on a plane but on a highly curved surface since the portion of the perceived luminance information that it carries increases in magnitude non-linearly with colour saturation (i.e. with distance from achromatic colours or grey scale).

In analogue systems, it has been widely believed that provided matching equations are used at the coder and decoder, it matters little what those equations are. In practice, the further the equations differ from the ideal, and the more the chroma signals are bandwidth limited, the more the luminance resolution loss becomes and the worse the chroma noise pollutes the luminance (hence the frequently visible and sometimes objectionable ‘lumpy’ low-frequency noise in VHS recording).

2.4 Other forms of coding

There are a number of alternative methods possible for signal coding. Of these, perhaps the one of most interest in the present context is ‘Constant Luminance’

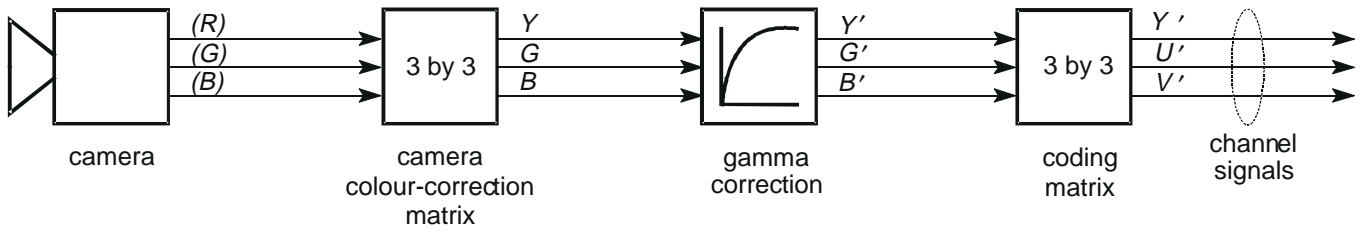


Fig. 5a - Constant-luminance coder.

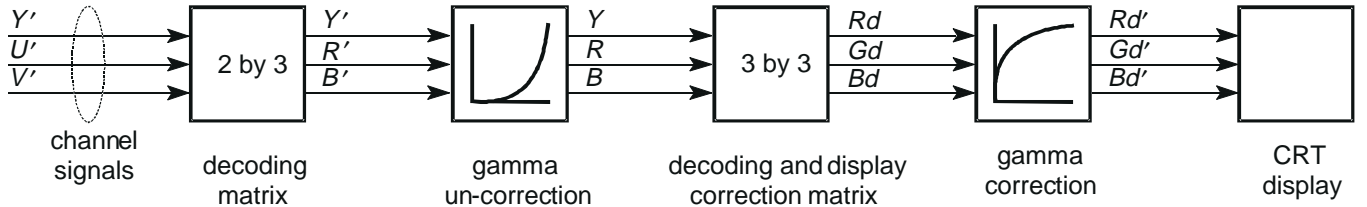


Fig. 5b - Constant luminance decoder.

coding. This coding scheme ensures that the perceived luminance is fully independent of the contents of the chroma signal and so the luma signal contains only luminance information. Thus the luma equation *must* be:

$$L' = (Y_R R + Y_G G + Y_B B)'$$

Chroma equations can then be generated in any convenient way, for example:

$$C_B = \alpha (B' - L') \quad \text{and} \quad C_R = \beta (R' - L')$$

or by using linear colour differences:

$$C_B = \alpha (B - L) \quad \text{and} \quad C_R = \beta (R - L)$$

It is worth noting that the use of colour *differences* in this way ensures that the signals are bipolar, and that this is of major significance to systems in which sub-carriers are used to carry the chroma signals. For component systems, where the luma and chroma signals are separated by time instead of by frequency, the bipolarity of the chroma signal is less important. Provided that the luminance information is wholly contained within the luma signal, then the coding system will have constant-luminance performance no matter how the chroma signals are formed. Chroma signals formed by differences as described still contain some luminance information but it is not used in the decoder, the only way to remove luminance from the chroma signals is to form them from chromaticity coordinates.

Fig. 5 shows a block diagram of a typical Constant-Luminance coder and decoder. In this instance, it is possible to make corrections for any differences between the 'camera primaries' and the *actual* display primaries; since it is necessary to decode the transmitted signals into the linear form ($Y R B$) in order to recover the G signal, it is possible to perform a linear matrix conversion such that the new drive signals ($R_d G_d B_d$) match the actual display.

Disadvantages of constant-luminance coding:

- The decoder is more complex than for conventional decoding. Non-linear circuitry is required in order to obtain the G signal from the luma and chroma signals.
- Clearly, the signals are not compatible with conventionally coded signals. Although it is possible to design a system which has a degree of compatibility, its performance with constant-luminance coding and conventional decoding would always be a compromise.

Advantages of constant-luminance coding:

- Luminance information is carried entirely in the luma signal, and so monochrome monitoring is accurate.
- Noise, quantisation effects and bit-rate reduction artefacts which afflict the chroma signals are observed only as hue and saturation effects, they do not pollute the perceived luminance. Thus it should be possible to compress the chroma signals more than with conventional equations.
- Luminance resolution is preserved since none of it travels via the reduced bandwidth chroma

channels. This effect can be quite dramatic. For example, only about 10% of the luminance information present in a fully saturated blue signal ($R = 0$, $G = 0$, $B = 1$) travels via the luma channel of a conventionally coded signal, and thus any luminance detail is attenuated by about 20 dB. The same applies for transmission of pure red, where detail is attenuated by about 11 dB.

- The coder can be more simple than for conventional coding, if a simple linear form of chroma signal is used. All that is required is that the luminance is formed from linear *RGB* signals, and then gamma corrected. The *RGB* signals can remain linear unless non-linear forms are required for chroma coding or for local monitoring.
- The ‘camera primaries’ can be chosen for the transmission of maximum colour gamut, since the decoder has access to the linear *RGB* signals and can, therefore, perform the necessary linear matrix conversion to derive correct drive voltages for the primaries actually used. Thus the transmission system designer makes no assumptions about the properties of the display and does not have to make allowance for its deficiencies. This is of great benefit in any environment in which many different types of display can be expected to be used. In the same way, the transfer function (voltage to light) of the actual display can be fully corrected.

Compromises are possible, in which some of the performance characteristics of a constant-luminance system are preserved on a modified conventional coder. Various techniques have been suggested for improving conventional coding, but all have involved extra circuitry in the coder and result in extra expense for the broadcaster.

An early example of this was the EMI four-tube camera, in which the luma signal was generated conventionally from non-linear *RGB* at frequencies within the chroma bandwidth, but was replaced with the signal from the fourth (luminance) tube at higher frequencies. This gave the camera the resolution performance advantages of constant-luminance coding without the complexity of the decoder. However, the luma signal was still polluted with chroma noise, and the camera was rather inefficient in its use of light, since a four-way dichroic splitter was needed.

3. TRANSCODING CODED SIGNALS

All transmitted signals are coded. The coding equations make assumptions about the display (its transfer characteristics and primaries). If the user’s display is different from the notional display which defines the transmission equations, then the user must perform some processing in order to maintain accurate colorimetry. It is up to the user to decide how to do this, or even whether to do it at all.

There are three distinct conditions which, in order of difficulty for dealing with, are:

- Different equations
- Different transfer characteristics
- Different display primaries

3.1 Different equations

This problem is simple to deal with; always use the correct equations. Use of incorrect equations will produce colour errors which can be calculated. For example, if a signal is coded according to ITU-R Rec. BT. 709 equations, and decoded using ITU-R Rec. BT. 601 equations, there can be errors of up to 20% in *RGB* signals.

If the coding is to be regarded as ‘reversible’, then the decoder must *always* use the inverse equations derived from the coding equations, whatever they may be. But note that true reversibility can only be achieved with a constant-luminance system, as only this gives no loss of perceived luminance resolution.

Fig. 6 shows a block diagram of a typical equation transcoder. It is possible to concatenate the two matrices into a single matrix, but this implementation shows exactly what is being done to the signals.

3.2 Different transfer characteristics

This problem is less easy to deal with, but failure to do so can result in very much larger colour errors than from use of the wrong decoding equations.

Suppose, for example, that the display is a linear device, i.e.: $Light = kV$, where k is simply a scaling factor. Clearly, the displayed colours will be wrong unless the transmitted transfer characteristic is removed. The simplest approach is to ignore the details of the transmitted transfer characteristic and merely assume that the source camera was set up to produce

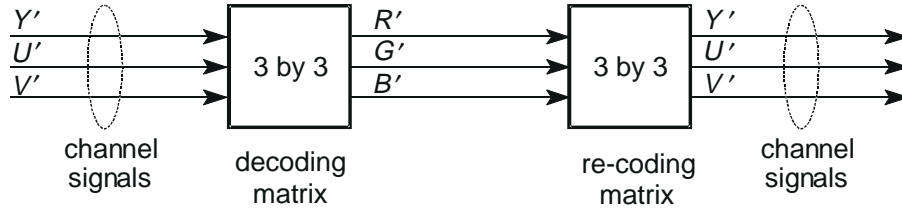


Fig. 6 - Coding equation transcoder.

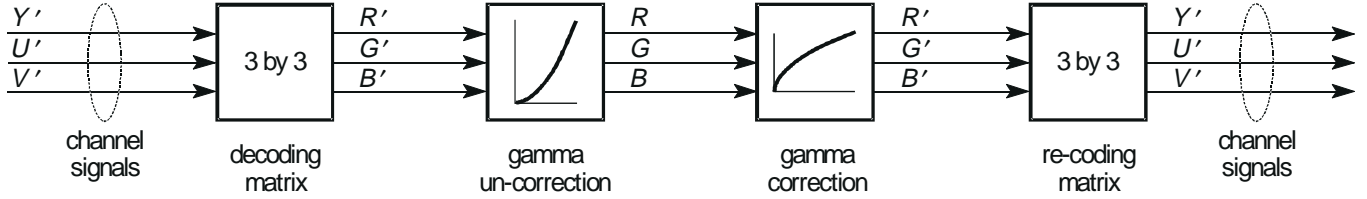


Fig. 7 - Transfer characteristic transcoder.

definitive pictures on a studio monitor (with a gamma (γ) of 2.3 to 2.4). The decoded *RGB* signals should then be passed through a mathematical process such as:

$$R = R'^{\gamma} \quad G = G'^{\gamma} \quad B = B'^{\gamma}$$

where γ is 2.3 or 2.4. In a digital decoder this can be simply a look-up table (possibly in ROM), although 12-bit linear scaling is needed for 8-bit non-linear signals.

It is more accurate, but more difficult and much more risky, to use the inverse of the transmission equation to return to the original linear values. For example, the following equations are a matching set:

$$V = (1 + a) \text{Light}^{\left(\frac{1}{\gamma}\right)} - a \quad \text{and} \quad \text{Light} = \left(\frac{V + a}{1 - a} \right)^{\gamma}$$

Using matching equations such as these, the linear camera *RGB* values can be recovered. The linear part of the curve must also be used, since such curves are supposed to be designed to allow for a tangential linear lower part. If the camera curve is specified as linear below $\text{Light} = c$, then the slope must be:

$$g = (1 + a) c^{\left(\frac{1}{\gamma}\right)} - a$$

since the linear and non-linear curves must meet at the break point. This implies that the specification should not include both the break point and the linear slope as parameters, since each is defined by the other. In the linear region:

$$V = g \text{Light} \quad \text{for } 0 \leq \text{Light} < c$$

and

$$\text{Light} = V/g \quad \text{for } 0 \leq V < cg$$

Use of these equations will reproduce the original *RGB* signals precisely. However, it is a great leap of faith to believe that all signals are generated with the specified camera curve, since camera manufacturers frequently provide several different gamma-correction curves, and an operator in a television studio can manipulate the black level and colour balances in order to arrive at a satisfactory picture. Thus it is much more reliable to define the source colour as that emanating from the camera operator's monitor.

If the display has some completely different type of characteristic from the classical CRT, then the display designer must make provision to correct for it, either by decoding to linear *RGB*, as shown above, or by making appropriate distortions to convert the camera law to match that of the display.

Fig. 7 shows a block diagram of a typical transcoder for changing transfer characteristics. Again, it is possible to concatenate the two non-linear curves into a single correction curve, but this implementation shows exactly what is being done to the signals. Indeed, in a digital system it is possible to combine all these operations (from $Y'U'V'$ in to $Y'U'V'$ out) into a single lookup table or ROM, albeit a very large one.

3.3 Different display primaries

This is by far the most difficult to deal with since some tricky mathematics is required.

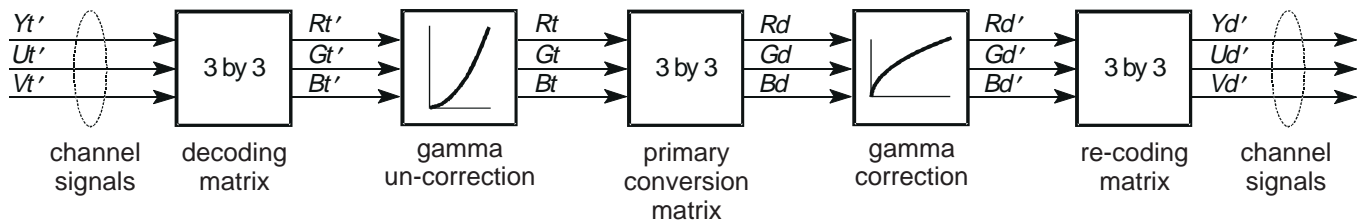


Fig. 8 - Primaries transcoder.

- The linear *RGB* signals must be recovered by using the correct decoding equations and non-linear transfer characteristics as described above, but it is best to ignore the actual camera characteristic and to derive the linear *RGB* light from the studio monitor.
- The display designer/user must then perform some arithmetic to establish the 3-by-3 linear matrix* which relates the tristimulus values for to the display ($R_d G_d B_d$) to those of the transmission ($R_t G_t B_t$). This involves knowledge of the chromaticity coordinates of the display, and those of the white point for which $R_t = G_t = B_t = 1$.
- Apply this transfer matrix to the decoded signals ($R_t G_t B_t$) to produce drive signals for the display ($R_d G_d B_d$) and distort them to match the electro-optic transfer characteristic of the display as described above.

Fig. 8 shows a block diagram for a typical transcoder to correct for different primaries. Note that the similarity between this operation and the typical constant-luminance decoder (Fig. 5). Once more, it is possible to combine parts or all of this operation into a single lookup table or ROM.

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* Appendix II gives the detailed procedure for calculating these matrices from the chromaticity coordinations of the primaries.

APPENDIX I

Basic Colorimetry

The principles of colorimetry, as applied in television and video systems, are those of metameric* colour-matching.**

Any colour can be described by three properties or signals, known as tristimuli, e.g. in television these could be *RGB* or *YC_rC_b*. Ideally they form an orthogonal set such that variations in one signal does not cause variations in the *perception* of the others. In practice, this is not always achieved, and can cause varying degrees of problem.

The fundamental description of a colour comes from its spectral properties. For an emissive colour (light bulb, the sky, candle flame, crt phosphor etc.) this is the emissivity (power distribution by wavelength). For a non-emissive colour (paper, cloth, scenery, stained glass, filters etc.) it is the transmissivity or reflectivity (attenuation by wavelength) multiplied (wavelength by wavelength) by the emissivity of the illuminant. To define a colour uniquely, it is necessary to know the spectral power distribution data (or the product of transmission or reflection with that of the illuminant) for the colour. This is measured most accurately by spectro-radiometry.

The spectral power data can be reduced to a set of tristimuli by multiplying the spectral ordinates (wavelength by wavelength) by each of the three CIE colour-matching functions† for the colour-normal observer. The colour-matching functions are shown in Fig. A1.1 and characterise the colour responsivity of the colour-normal observer (the average non colour-blind viewer). When passed through the camera matrix equation (see later), the colour-matching functions produce the ideal taking-characteristics for the camera, such as are shown in Fig. A1.1. The tristimuli are the integrals (areas under the curves) of the resultant multiplication:

$$X = \sum_{380}^{760} P_{\lambda} \frac{-}{x_{\lambda}} d\lambda \quad Y = \sum_{380}^{760} P_{\lambda} \frac{-}{y_{\lambda}} d\lambda \quad Z = \sum_{380}^{760} P_{\lambda} \frac{-}{z_{\lambda}} d\lambda$$

where P_{λ} is the spectral data for the colour (multiplied by that of its illuminant if it is not self-emissive), $\bar{x} \bar{y} \bar{z}$ are the CIE colour-matching functions† (strictly, this is not an integration, but a summation, because the spectral data curves are held as tables, sampled at regular intervals, such as 5 or 10 nm). The integration is usually done from 380 to 760 nm (nanometers) which covers all the visible part of the spectrum. The resulting values XYZ are the tristimuli for the colour in the CIE 1931 XYZ system. (Note that this is valid only for the average colour-normal observer, about 14% of males and 2% of females will

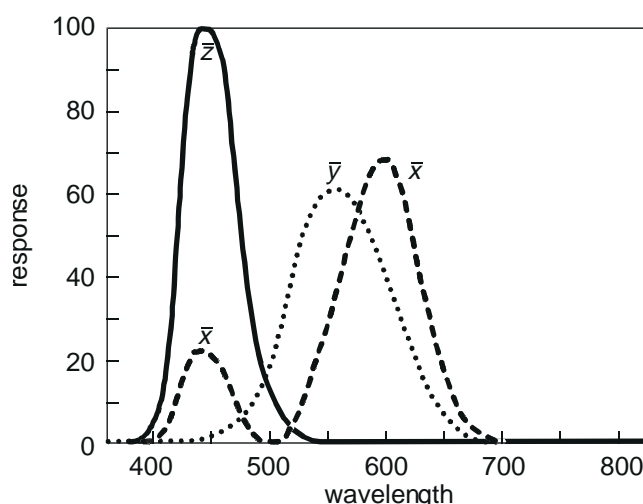


Fig. A1.1 - CIE colour matching functions.

* A *metameric* colour match exists between two colours which look the same, but have different spectral characteristics, while an *isomeric* colour match exists between colours which not only look the same but have identical spectral properties.

** Colour-matching means that colours look the same as each other to the colour-normal observer. All the equations used in this Appendix relate to colour-matching and not to mathematical identities.

† The CIE colour-matching functions are tabulated in CIE publication Tech. 15, 1971.

disagree with the results because their colour vision is abnormal in some way.)

In physical terms, this process of multiply and integrate is a colour-matching operation. At each wavelength, the values $\bar{x} \bar{y} \bar{z}$ are the amounts of the XYZ primaries needed for an average colour-normal observer to match an equi-energy colour (i.e. a colour with no spectral variation). The integration adds together all the components across the spectrum to give a set of three values which uniquely describes the colour. The actual primaries XYZ were chosen by the CIE such that they could quantify all visible colours using only positive values; this means that the primaries themselves must be un-real colours since they lie outside the spectrum boundary and are therefore invisible. This raises no mathematical problem, only one of understanding.

The value Y is the luminance of the colour, by definition. The values of XYZ are usually scaled such that $Y = 1$ for peak signal at the white or balance point (D_{65} in television).

Since the luminance is quantified by Y , only two more variables are needed to describe the colour properties and these are called *chromaticity coordinates*. In the CIE 1931 XYZ system any pair of values from the set of xyz is acceptable:

$$x = \frac{X}{X + Y + Z} \quad y = \frac{Y}{X + Y + Z} \quad z = \frac{Z}{X + Y + Z}$$

since $x + y + z = 1$. By tradition, x and y are normally used. Thus a typical specification for a colour would use values for Y, x and y . The equations are linear, and so can be inverted:

$$X = \frac{x}{y} Y \quad Z = \frac{z}{y} Y = \frac{(1 - x - y)}{y} Y$$

There are many other colour systems in use in the colour industry but the only ones of major significance to the television industries are the CIE 1931 XYZ and the CIE 1976 $U'V'W'$ systems. Conversion between these systems is straightforward:

$$u' = \frac{4x}{12y - 2x + 3} \quad v' = \frac{9y}{12y - 2x + 3} \quad \text{and} \quad x = \frac{4.5u'}{3u' - 8v' + 6} \quad y = \frac{2v'}{3u' - 8v' + 6}$$

The chromaticity coordinates of primaries are freely quoted in either of these systems, and this leads to a problem of minor significance. For example, the ITU-R Rec. BT. 624 (PAL systems) primaries are specified precisely to two decimal places in xy values (e.g. for red, $x = 0.640000\dots$), but are frequently found expressed to three places in $u'v'$ values. Since the $u'v'$ values were calculated from the xy values (the definitions) and then rounded to three decimal places, it follows that the $u'v'$ values are less accurate and so should not be used in precise system calculations.

ITU-R Rec. BT. 624 Chromaticity Coordinates					
	x	y	z	u'	v'
red	0.64	0.33	0.03	0.451	0.523
green	0.29	0.6	0.11	0.121	0.561
blue	0.15	0.06	0.79	0.175	0.158
white (D_{65})	0.3127	0.3290	0.3583	0.1978	0.4683

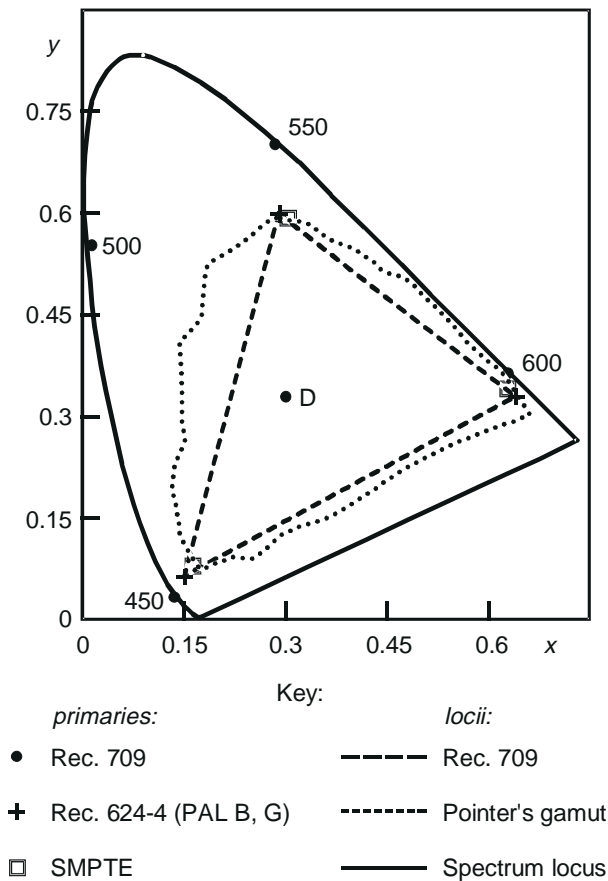


Fig. A1.2 - CIE 1931 xy chromaticity problem.

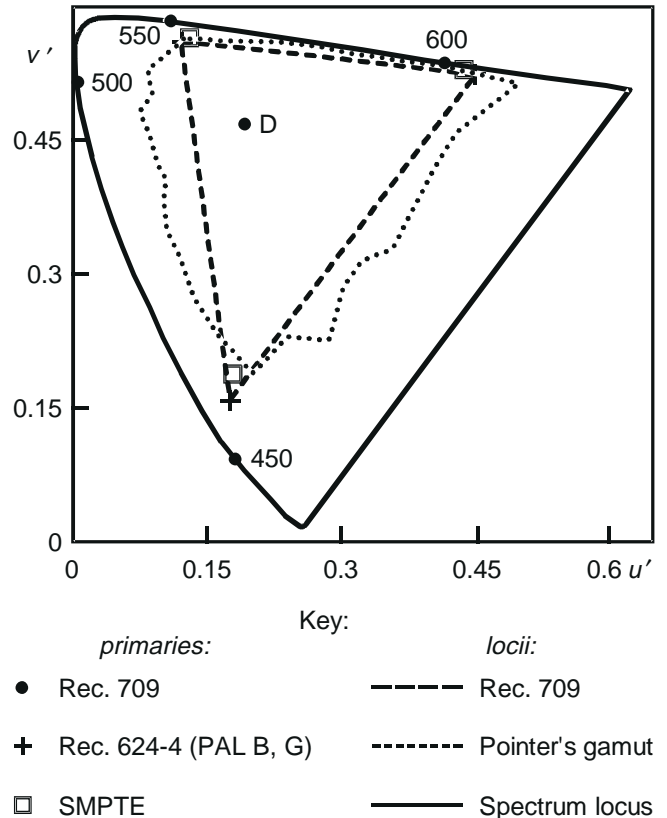


Fig. A1.3 - CIE 1976 u'v' chromaticity diagram.

This is generally true for all chromaticity and tristimulus calculations; data values should never be truncated or rounded in calculation. Similarly, the values for D_{65} are rounded to three or four decimal places and these should not be used for conversion into other colour spaces. The chromaticity coordinates of D_{65} can be calculated from first principles, or tabulated values are available, both can be found in CIE Tech.15,⁵ 1971 ($x = 0.312713$, $y = 0.329016$).

Figs. A1.2 and A1.3 are chromaticity diagrams in the CIE 1931 XYZ and CIE 1976 U'V'W' spaces. Both diagrams show the locations of the ITU 709 primaries, the spectrum locus (any point on which is the most saturated visible colour at that hue), the locus of Planckian radiators (black bodies, tungsten lamps etc.), and a projection of Pointer's Real Surface Colours Gamut. Pointer's gamut⁶ is a three dimensional representation of the most saturated surface (i.e. non self-luminous) colours measured at varying values of luminance and hue; it represents the body of real colours which any video transmission system should be able to code and reproduce without serious distortion.

The 1931 xy diagram (Fig. A1.2) is rarely used by television colorimetrists because equally visible colour differences in it are not represented by equal magnitude vectors. In an ideal chromaticity diagram, all vectors of equal length (each of which points from one colour to another of equal luminance) would represent equally visible colour differences. A unit length vector in the xy diagram varies in visibility by about 40:1. In the u'v' diagram (Fig. A1.3) a unit vector varies in visibility by about 7:1. No-one has yet managed to produce a colour space, into which the XYZ tristimuli can be linearly transformed, in which unit vectors represent uniformly visible colour differences. That is the Holy Grail of colorimetry.

APPENDIX II

Television Colorimetry

Calculations on the colorimetry of television systems require knowledge of the matrix equations which connect the television primaries with those of the CIE 1931 XYZ system. The mathematics is fairly complex but involves no difficult processes. The mathematics must produce matrix equations of the form:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

which, respectively, define the camera linear output signals (ie. before gamma-correction) in terms of the CIE tristimuli of the scene, and the displayed colour in terms of the linear drive signals (ie. cathode-ray tube beam current). The a and b matrices are each the inverse of the other. The coefficients of the a and b matrices are each the tristimuli of the primaries in the right-hand 3-by-1 matrix expressed in the colour space of the primaries in the left-hand side 3-by-1 matrix. The problem is how to find the matrix coefficients, since we are not given the tristimuli of the primaries, only their chromaticity coordinates.

These matrix equations are statements of the additive nature of colour mixing, that any colour can be matched by a mixture of any three other colours provided that none of the three can be made by mixing the other two. This is an expression of one of Grassman's laws.⁷

The primary luminances can be found in two ways. The most obvious direct way is to construct a display with these primaries, and colour balance it by adjusting the RGB drives to produce the requisite white colour. Each of the primaries will then have been adjusted to have the correct luminance. Then by switching on each of the (balanced) primaries in turn, the luminance can be measured using an accurate luminance meter. The problem with this method is that the accuracy of the white point and the measured luminance is only as good as the available measuring equipment. For metrology reasons, chromaticities are much easier to measure accurately than luminances and so it makes sense to adopt a mathematical solution, rather than relying on dubious measurements.

The following calculations show the process of matrix calculation for the ITU-R Rec. BT. 709 primaries. Also the same process is used to calculate the matrices connecting ITU-R Rec. BT. 709 with ITU-R Rec. BT. 624 primaries (PAL).

Colour	Chromaticity, system ITU 709		Chromaticity, system ITU 624	
	x	y	x	y
red	0.640	0.330	0.64	0.33
green	0.300	0.600	0.29	0.60
blue	0.150	0.060	0.15	0.06
white (D ₆₅)	0.3127	0.3290	0.313	0.329

The equation of the ITU-R BT.709 display is defined by the tristimuli of the primaries:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{R_{709}} & X_{G_{709}} & X_{B_{709}} \\ Y_{R_{709}} & Y_{G_{709}} & Y_{B_{709}} \\ Z_{R_{709}} & Z_{G_{709}} & Z_{B_{709}} \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

but the tristimuli are not yet known, so the equation is rewritten using the chromaticity coordinates and three scaling factors k which relate chromaticity value to tristimulus value. The scaling factors can be written as a diagonal matrix, but that is not essential for a complete understanding of the mathematics:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} k_{R_{709}} x_{R_{709}} & k_{G_{709}} x_{G_{709}} & k_{B_{709}} x_{B_{709}} \\ k_{R_{709}} y_{R_{709}} & k_{G_{709}} y_{G_{709}} & k_{B_{709}} y_{B_{709}} \\ k_{R_{709}} z_{R_{709}} & k_{G_{709}} z_{G_{709}} & k_{B_{709}} z_{B_{709}} \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

The balance condition is now invoked, since it defines the values of XYZ and RGB for one colour:

$$\begin{bmatrix} X_{D_{65}} \\ Y_{D_{65}} \\ Z_{D_{65}} \end{bmatrix} = \begin{bmatrix} k_{R_{709}} x_{R_{709}} & k_{G_{709}} x_{G_{709}} & k_{B_{709}} x_{B_{709}} \\ k_{R_{709}} y_{R_{709}} & k_{G_{709}} y_{G_{709}} & k_{B_{709}} y_{B_{709}} \\ k_{R_{709}} z_{R_{709}} & k_{G_{709}} z_{G_{709}} & k_{B_{709}} z_{B_{709}} \end{bmatrix} \begin{bmatrix} R_{709 D_{65}} \\ G_{709 D_{65}} \\ B_{709 D_{65}} \end{bmatrix}$$

Substituting the numerical values for the ITU-R BT. 709 system:

$$\begin{bmatrix} 0.3127 / 0.3290 \\ 1 \\ (1 - 0.3127 - 0.3290) / 0.3290 \end{bmatrix} = \begin{bmatrix} k_{R_{709}} 0.64 & k_{G_{709}} 0.30 & k_{B_{709}} 0.15 \\ k_{R_{709}} 0.33 & k_{G_{709}} 0.60 & k_{B_{709}} 0.06 \\ k_{R_{709}} (1 - 0.64 - 0.33) & k_{G_{709}} (1 - 0.30 - 0.60) & k_{B_{709}} (1 - 0.15 - 0.06) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

This equation can be solved since the only unknowns are the three k values, using either conventional simultaneous equation arithmetic or matrix manipulation. The solutions are:

$$k_{R_{709}} = 0.6444, \quad k_{G_{709}} = 1.1919, \quad k_{B_{709}} = 1.2030$$

Applying these multipliers to the chromaticity coordinates yields the display equation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9504 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

and by inverting this matrix we can arrive at the camera matrix:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.2408 & -1.5373 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0571 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

By an identical process, the equations for ITU-R. Rec. BT. 624 (PAL systems) are:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4306 & 0.3416 & 0.1783 \\ 0.2220 & 0.7067 & 0.0713 \\ 0.0202 & 0.1296 & 0.9392 \end{bmatrix} \begin{bmatrix} R_{624} \\ G_{624} \\ B_{624} \end{bmatrix}$$

and:

$$\begin{bmatrix} R_{624} \\ G_{624} \\ B_{624} \end{bmatrix} = \begin{bmatrix} 3.0632 & -1.3933 & -0.4758 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0679 & -0.2288 & 1.0693 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

The connecting matrices between these two systems can be calculated by matrix multiplication:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.2408 & -1.5373 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0571 \end{bmatrix} \begin{bmatrix} 0.4306 & 0.3416 & 0.1783 \\ 0.2220 & 0.7067 & 0.0713 \\ 0.0202 & 0.1296 & 0.9392 \end{bmatrix} \begin{bmatrix} R_{624} \\ G_{624} \\ B_{624} \end{bmatrix}$$

which yields:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 1.0440 & -0.0440 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0118 & 0.9882 \end{bmatrix} \begin{bmatrix} R_{624} \\ G_{624} \\ B_{624} \end{bmatrix}$$

and its inverse:

$$\begin{bmatrix} R_{624} \\ G_{624} \\ B_{624} \end{bmatrix} = \begin{bmatrix} 0.9578 & 0.0422 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & -0.0119 & 1.0119 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

and it is these two matrices which must be used to convert linear *RGB* values between the two systems.

Identical processing can be used to solve the equations for any set of primaries, and to calculate the inter-connecting matrices between any pair of systems.

Although calculated values have been given here to only four decimal places, the calculations were actually much more precise. Premature rounding of values can lead to surprisingly large errors in matrix calculations. All calculations must be done to at least *ten* decimal places.

